# Project IV 2022/23

## Simulation of conditioned diffusion processes

Stochastic differential equations (SDEs) can be used to model many continuous time processes such as stock price, predator-prey dynamics, cellular processes and epidemics. For example, the well known Black-Scholes model assumes that stock price follows a geometric Brownian motion, which can be formulated as an SDE and used as the basis of more realistic models.

The solution of a multivariate, nonlinear SDE is rarely tractable in closed form, necessitating the use of numerical methods such as the Euler-Maruyama approximation. Under such a scheme, forward simulation is straightforward yet efficiently sampling the process at times between two known fixed values remains problematic. This project will seek efficient discrete-time approximations of conditioned diffusion processes, typically termed diffusion bridges, and use these to simulate sample paths of a number of conditioned processes of interest.

The ability to simulate an end-point conditioned diffusion process is a key ingredient of simulation-based Bayesian inference schemes such as Markov chain Monte Carlo, which target the joint posterior density of the SDE parameters and latent values between observation instants. The inference methodology can be tested in a variety of settings, using real and synthetic data, depending on the interests of the project student.

**Prerequisites**: Bayesian Statistics III and either Stochastic Processes III or Financial Mathematics III. Familiarity with the statistical language R is essential.

### Resources (indicative)

- Searching for ``Brownian bridge'' will give lots of info on conditioned Brownian motion.
- Fuchs, C. (2013). Bayesian inference for diffusion processes with applications in life sciences. Springer. https://edoc.ub.uni-muenchen.de/12136/1/Dargatz_Christiane.pdf
- A fairly recent survey of bridge constructs: https://arxiv.org/pdf/1509.09120.pdf

### Supervisor

Andy Golightly (andrew.golightly@durham.ac.uk)